NAG Fortran Library Routine Document

F07ANF (ZGESV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07ANF (ZGESV) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07ANF (N, NRHS, A, LDA, IPIV, B, LDB, INFO)
INTEGER            N, NRHS, LDA, IPIV(*), LDB, INFO
complex*16         A(LDA,*), B(LDB,*)
```

The routine may be called by its LAPACK name ***zgesv***.

3 Description

The LU decomposition with partial pivoting and row interchanges is used to factor A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: N – INTEGER *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $N \geq 0$.

2: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $NRHS \geq 0$.

3: A(LDA,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, N)$.

On entry: the n by n coefficient matrix A.

On exit: the factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07ANF (ZGESV) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

5: IPIV($*$) – INTEGER array *Output*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On exit: if INFO ≥ 0, the pivot indices that define the permutation matrix P; at the ith step row i of the matrix was interchanged with row IPIV(i). IPIV(i) $= i$ indicates a row interchange was not required.

6: B(LDB,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r matrix of right-hand side matrix B.

On exit: if INFO $= 0$, the n by r solution matrix X.

7: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07ANF (ZGESV) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

8: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, u_{ii} is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies the equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1}$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of F07ANF (ZGESV), F07AVF (ZGERFS) can be used to estimate the condition number of A and F07AVF (ZGERFS) can be used to obtain approximate error bounds. Alternatives to F07ANF (ZGESV), which return condition and error estimates directly are F04CAF and F07APF (ZGESVX).

8 Further Comments

The total number of floating-point operations is approximately $\frac{8}{3}n^3 + 8n^2r$, where r is the number of right-hand sides.

The real analogue of this routine is F07AAF (DGESV).

9 Example

To solve the equations

$$Ax = b,$$

where A is the general matrix

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}$$

and

$$b = \begin{pmatrix} 26.26 + 51.78i \\ 6.43 - 8.68i \\ -5.75 + 25.31i \\ 1.16 + 2.57i \end{pmatrix}.$$

Details of the LU factorization of A are also output.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F07ANF Example Program Text
*     Mark 21 Release. NAG Copyright 2004.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
      INTEGER          LDA
      PARAMETER        (LDA=NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
*     .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), B(NMAX)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         X04DBF, ZGESV
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07ANF Example Program Results'
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
```

```
      IF (N.LE.NMAX) THEN
*
*        Read A and B from data file
*
         READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
         READ (NIN,*) (B(I),I=1,N)
*
*        Solve the equations Ax = b for x
*
         CALL ZGESV(N,1,A,LDA,IPIV,B,N,INFO)
*
         IF (INFO.EQ.0) THEN
*
*           Print solution
*
            WRITE (NOUT,*) 'Solution'
            WRITE (NOUT,99999) (B(I),I=1,N)
*
*           Print details of factorization
*
            WRITE (NOUT,*)
            IFAIL = 0
            CALL X04DBF('General',' ',N,N,A,LDA,'Bracketed','F7.4',
     +                  'Details of factorization','Integer',RLABS,
     +                  'Integer',CLABS,80,0,IFAIL)
*
*           Print pivot indices
*
            WRITE (NOUT,*)
            WRITE (NOUT,*) 'Pivot indices'
            WRITE (NOUT,99998) (IPIV(I),I=1,N)
         ELSE
            WRITE (NOUT,99997) 'The (', INFO, ',', INFO, ')',
     +        ' element of the factor U is zero'
         END IF
      ELSE
         WRITE (NOUT,*) 'NMAX too small'
      END IF
      STOP
*
99999 FORMAT ((3X,4(' (',F7.4,',',F7.4,')',:)))
99998 FORMAT (1X,7I11)
99997 FORMAT (1X,A,I3,A,I3,A,A)
      END
```

9.2 Program Data

```
F07ANF Example Program Data

   4                                                          :Value of N

 (-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
 (-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
 (-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
 ( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67)  :End of matrix A

 (26.26,51.78) ( 6.43,-8.68) (-5.75,25.31) ( 1.16, 2.57)  :End of vector b
```

9.3 Program Results

```
F07ANF Example Program Results

Solution
   ( 1.0000, 1.0000) ( 2.0000,-3.0000) (-4.0000,-5.0000) ( 0.0000, 6.0000)

Details of factorization
                   1                  2                  3                  4
1  (-3.2900,-2.3900) (-1.9100, 4.4200) (-0.1400,-1.3500) ( 1.7200, 1.3500)
2  ( 0.2376, 0.2560) ( 4.8952,-0.7114) (-0.4623, 1.6966) ( 1.2269, 0.6190)
3  (-0.1020,-0.7010) (-0.6691, 0.3689) (-5.1414,-1.1300) ( 0.9983, 0.3850)
4  (-0.5359, 0.2707) (-0.2040, 0.8601) ( 0.0082, 0.1211) ( 0.1482,-0.1252)

Pivot indices
          3          2          3          4
```